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MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

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A meeting of the Association for Symbolic Logic was held at the Wisconsin Center, University of Wisconsin, Madison, on April 16–17, 1982, in conjunction with a meeting of the American Mathematical Society. There were four invited hour addresses: Ward Henson, *Banach space model theory*, David Kueker, *Some model-theoretic conjectures and stability*, Menachem Magidor, *Countable unions of constructible sets*, and John Steel, *Determinacy in the Mitchell models*. There were twelve contributed lectures and two contributed papers presented by title. The abstracts follow.

H. JEROME KEISLER, Chairman

IRVING H. ANELLIS, *Formal arithmetic and the definition of number.*

The attempt to construct arithmetic within a formal logical system dates to Frege. The attempt by Peano [5] to axiomatize arithmetic is weak, since the Peano axiom system fails to provide an inference rule for deriving formulas (see [6]). Without an inference rule, the arithmetic formulas in the Peano system can merely be listed, and are not obtained logically; worse, introduction into Peano arithmetic of inference rules and strong mathematical induction leads to an inconsistency (see [1]). The systems of Frege [2], [3], Zermelo [9], and Whitehead and Russell [7] are rigorous formal systems in which arithmetic can be constructed by introduction into the syntax of first-order functional calculus of a set of precisely defined operators (addition, multiplication, identity) and functions (number, and number-generators such as successor and ancestral and proper ancestral), and at least one constant (zero), while in contrast Peano's system presents a recursive definition (Definition 10 of [5]) of *number* rather than a number-generator. Each system (but specifically Frege's and Whitehead-Russell's) becomes more felicitous if a second-order functional calculus is used to develop arithmetic, although familiar difficulties, most notably the Russell Paradox, obtain for a second-order functional calculus whose semantic interpretation is extensionalistic (whether Fregean *Wertverlauf* or Russellian set-theoretic). We will not dwell upon these difficulties, but rather provide the formalization of arithmetic employing the axioms of Zermelo.

With a construction of Zermelo arithmetic, the formalization of intuitive arithmetic as primitive recursive arithmetic completed, we examine the formalizations of Frege and Russell, on the basis of which it can clearly be shown that such authors as Hambourger [4] who speak of the "Frege-Russell definition of number" are guilty of conflating two quite distinct constructions. Finally, elucidating the set-theoretic semantic of Russell, it is shown that the claim by Wittgenstein [8], that such statements as 'two is a number' are ill-formed and consequently incapable, as an "illegitimate totality", of serving as a wff in the system, is erroneous.

Dedicated to Jean van Heijenoort's Seventieth Birthday.

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PAUL BANKSTON, *Semantic preservation by functors of algebras of continuous functions*.

We define three first order languages L_T, L_C, L_B as follows: L_T has predicate symbols for points, sets, and membership (there is a natural semantic relationship (z-satisfaction) between topological spaces and L_T-sentences); L_C has symbols representing all continuous finitary operations on the real line (it is appropriate to consider sets $C^*(X)$ of bounded continuous real-valued functions defined on the space X to be L_C-algebras); and L_B is simply the language of Boolean algebras (we let $B(X)$ denote the Boolean algebra of clopen subsets of X).

Employing three distinct "ultrapower" constructions, we prove the following.

THEOREM 1. *Let X and Y be Tichonov spaces satisfying the same L_T-sentences. Then $B(X)$ and $B(Y)$ satisfy the same L_B-sentences, and $C^*(X)$ and $C^*(Y)$ satisfy the same positive universal L_C-sentences.* (*The converses are false.*)

THEOREM 2. *Let X and Y be strongly 0-dimensional Tichonov spaces. If $B(X)$ and $B(Y)$ satisfy the same L_B-sentences then $C^*(X)$ and $C^*(Y)$ satisfy the same positive universal L_C-sentences.*

M. BICKFORD and C. F. MILLS, *Lowness properties for r.e. sets*.

Notation. A is an r.e. set. D_u is the finite set with canonical index u. $\{\sigma_e: e \in \omega\}$ is an effective list of truth table (tt-) conditions. We write $A \leq_{tt}^{B} C$ if there is a B-recursive function f such that $\forall n\, (n \in A \leftrightarrow C \models \sigma_{f(n)})$. If $B = 0$ we write just $A \leq_{tt} C$.

DEFINITION. The *A-correct content of W* (we write $W|A$) is $W \cap \{u: D_u \cap A = \emptyset\}$.

DEFINITION. A is low if $A' \leq_T 0'$ equiv. $A'' \leq_1 0''$. A is low$_2$ if $A'' \leq_T 0''$.

DEFINITION. S is an *e-oracle set for A* if S is finite and for some $x \in S$, $W_e|A \subseteq W_x$ and if $W_e|A$ is finite then W_x is finite.

We say *A has singleton* (*resp.* Δ_0^0, Σ_1^0, Σ_2^0) *oracle sets* if there is a recursive function, f, such that for every e, $\{f(e)\}$ (resp. $D_{f(e)}$, $W_{f(e)}$, $W_{f(e)}^K$) is an e-oracle set for A.

THEOREM 1. (a) (Soare) $A'' \leq_1 0'' \leftrightarrow$ *A has singleton oracle sets.*

(b) $A'' \leq_{tt} 0'' \leftrightarrow$ *A has Δ_0^0 oracle sets.*

(c) $A'' \leq_{tt}^{0'} 0'' \leftrightarrow$ *A has Σ_1^0 oracle sets.*

(d) $A'' \leq_T 0'' \leftrightarrow$ *A has Σ_2^0 oracle sets.*

REMARK. (a) and (d) characterize low and low$_2$ sets respectively. We call sets satisfying (c) *sober* and those satisfying (b) *depressed*. These classes are all distinct.

THEOREM 2. *Low $\subsetneqq$ depressed $\subsetneqq$ sober $\subsetneqq$ Low$_2$.*

In the proof of Theorem 2 we use the relativization of

THEOREM 3. *The following are equivalent.*

(a) $A' \leq_{tt} 0'$ (*we call such sets abject*).

(b) *There are recursive f, g such that for every e $W_{f(e)} \subseteq W_e$ and $|W_{f(e)}| < g(e)$ and if $W_e | A \neq \emptyset$ then $W_{f(e)} | A \neq \emptyset$.*

We have many results concerning abject sets. For example,

THEOREM 4. (a) *There are abject sets A and B such that $K \leq_T A \oplus B$.*

(b) *If A is abject, and B r.e. and $K \leq_{wtt} A \oplus B$, then $K \leq_{wtt} B$.*

Finally, we have the following jump interpolation theorem.

THEOREM 5. *If $B \leq_T C$ are r.e. sets and X is a set such that $B' \leq_T X \leq_1 C'$, then there is an r.e. set A such that $B \leq_T A \leq_T C$ and $X \leq_{tt} A' \leq_T X$.*

G.M. BRENNER, *A technique for constructing Boolean algebras from trees, so that properties of the trees are inherited.*

Given the well-founded tree $\langle T, \leq_T\rangle$, we let for all $t \in T$, $S_t^T = \{u \in T: u \geq_T t\}$; then define the *tree algebra generated by* T, B_T, as the closure of $\{S_t^T: t \in T\}$ under finite set unions and complementation relative to the set T. It is interesting to note that B_T inherits a number of the properties of T.

THEOREM 1. *For all $t \in T$ we let A_t denote the number of immediate successors to t in T. If a tree T satisfies* A. *height $T = \omega$,* B. *for all $t \in T$,* card *A_t is regular and uncountable, and* C. *for all distinct $u, t \in T$,* card $A_u \neq$ card A_t, *then B_T is rigid (i.e. has no nonidentity automorphisms).*

THEOREM 2 (BRENNER AND MONK). *If K is a regular, uncountable cardinal and T does not have a (well-ordered) chain of length K, then B_T does not have a well-ordered chain of length K.*

THEOREM 3. *If T_1 embeds in T_2, then B_{T_1} embeds in B_{T_2}.*

THEOREM 4. *Given tree T, we let T^* denote the tree extending T by the addition of a node at the end of each partial branch C (C is a partial branch iff C is a subset of a branch satisfying $u \in C \wedge t \leq u \Rightarrow t \in C$) of limit length. The Stone space of B_T is isomorphic to the set T^* under the topology with basis* $\{S_t^{T^*} \sim \sum_{\beta \in J} S_\beta$: J is a finite subset of T and $t \in J\}$.

We have investigated the closure properties of the class of tree algebras under homomorphic image, substructure and other operations as well as the relation of tree algebras to interval algebras. These topics will be covered in other papers.

MIKE CANJAR, *Model-theoretic properties of countable ultraproducts without CH.*

We examine model-theoretic properties of U-Prod N where U is a nonprincipal ultrafilter on ω, and N is the structure ω together with all its finitary functions and relations. This structure is ω_1-saturated, hence saturated if CH holds. We examine what can occur in models of ZFC + $\neg$CH.

Cohen model. Add k Cohen reals to a model of ZFC + GCH. We show that for all regular uncountable cardinals $a, b \leq k$ there exist ultrafilters $U_{a,b}$ so that the nonstandard part of U-Prod(ω, <) has coinitiality = a and cofinality = b. We show how to make these ultrafilters selective if $a = b$. Alternatively these ultrafilters can be constructed so that U-Prod N has no least sky. For $a = b = k$ we can construct the ultrafilter so that U-Prod N is saturated. These ultrafilters can be amalgamated in order to obtain infinitely many nonisomorphic ultraproducts of (ω, <). When $k > \omega_\omega$, there will be continuum many of these. Moreover when $k \geq \omega_3$ we can get continuum many ultrafilters whose ultraproducts of (ω, +, $*$) are nonisomorphic. (Under CH, all such ultraproducts are isomorphic.)

Random real model. Add k random reals to a model of ZFC + GCH. It is easy to see that all ultraproducts in this model will have cofinality = ω_1, which precludes saturation. We prove the existence of ultraproducts with the following saturation property: they consist of a saturated model of the theory of N plus a top sky. The properties of these ultrafilters are discussed. This seems to be the maximal amount of saturation possible in this model: There are Dedekind cuts in the top sky of every ultraproduct where the cofinality of the lower segment and the coinitiality of the upper segment are both ω_1. Also we show that in this model there are, for any uncountable regular a, ultrafilters U so that a is the coinitiality of the nonstandard part of U-Prod(ω, <).

JOHN CASE, KEMAL EBCIOGLU and MARK FULK, *R.e. inseparable general and subrecursive index sets.*

For A and B disjoint sets, A is *r.e. inseparable from B* iff every r.e. superset of A meets B. A is *effectively r.e. inseparable from B* iff one can recursively find from any r.e. index x, a counterexample to the separation of A from B by W_x.

On the general recursive level two main theorems are obtained. The first provides a strong sufficient condition for one index set to be effectively r.e. inseparable from another. It is proved by Kleene's Parametric Recursion Theorem and actually yields a characterization if the first index set is for a singleton class. It also immediately implies Rice's Theorem, its extensions including the Rice-Shapiro-Myhill-McNaughton Theorem [Ro 67], and improvements of the so-called relative solvability results of Rogers [Ro 67, p. 44]. The second theorem provides a stronger sufficient condition, but for not necessarily effective, r.e. inseparability. This theorem is proved by the k-ary recursion theorem.

Also examined are various subrecursive analogs of the above theorems. These are somewhat in the spirit of Kozen's subrecursive version of Rice's Theorem [Ko 80]. Here the subrecursive forms of the appropriate recursion theorems are employed: the recursion theoretic methods descend into the subrecursive at least down to reasonable indexings for *simultaneous* linear time and log space [RC 82].

Finally concrete analysis of a special case of a subrecursive analog casts serious doubt on an informal philosophical argument of Putnam [Pu 80, pp. 290–291] that the syntax of a certain logical language possesses an *intrinsic* fast, short grammar or decision procedure which parallels the corresponding truth definition.

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J.C.E. DEKKER, *Isols and balanced block designs with* $\lambda = 1$.

The word "*number*" stands for nonnegative integer, "*set*" for collection of numbers and "*class*" for collection of sets. A *BBD* on a finite set σ of cardinality ≥ 2 is a class Γ of subsets of σ (called *blocks*) for which there exist positive numbers k, r, λ such that $k \geq 2$ and (i) all blocks have cardinality k, (ii) every element of σ occurs in exactly r blocks, and (iii) every two distinct elements of σ occur together in exactly λ blocks. The numbers $v = \text{card}\,\sigma$, $b = \text{card}\,\Gamma$, k, r and λ are the *parameters* of Γ. The basic relations between the parameters of a BBD are: $bk = vr$ and $r(k-1) = \lambda(v-1)$. Using partial recursive functions we generalize the notion of a BBD on a finite set to that of an ω-BBD on an isolated set. We then prove $BK = VR$ and $R(K-1) = \lambda(V-1)$, where V, B, K, R are isols instead of numbers, while λ remains finite. As examples we discuss the cases $K = 3$, $\lambda = 1$ (Steiner triple systems) and $V = B$, $K = R$, $\lambda = 1$ (projective planes). Let c denote the cardinality of the continuum. While there are only denumerably many BBDs on finite sets, there are c ω-BBDs on isolated sets. Among these there are c Steiner triple systems (whose orders need not be $\equiv 1$ or 3 modulo 6) and c projective planes.

STEVE GRANTHAM, *Galvin's tree game.*

Let G be the class of trees without infinite branches. For S, T in G, the *Galvin game* $(S\colon T)$ is played as follows: a white pawn is placed at the root of S, a black pawn at the root of T, and players white and black alternately move *either* pawn from the node it is on to any immediate successor node; the winner is that player whose pawn reaches a terminal node ("queens") first. White moves first. Galvin showed white wins $(T\colon T)$ for any T. We extend this result to the case in which black is given infinitely many copies of the tree by defining valuation functions on the nodes of these trees which enable an explicit strategy to be given. We also define an equivalence relation on G and a well-ordering of the equivalence classes in terms of Galvin's game and use the valuation functions to study the properties of this well-ordering, e.g., there are only $\aleph_1$ equivalence classes of countable trees, even though there are $2^{\aleph_0}$ nonisomorphic countable trees (no matter how large $2^{\aleph_0}$ is). Furthermore, there is a countable tree whose predecessors in the well-ordering have order type ON (the type of the ordinals).

MATT KAUFMANN, *Blunt and topless end extensions.*

If $\mathfrak{A}$ is a model of ZFC, say that cof $(\mathfrak{A}) > \omega$ if its class of ordinals has uncountable cofinality.